

A blurred background image of a person wearing a yellow vest and hard hat, working on a computer keyboard in an industrial or construction setting.

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A Level

Computer Science

H446 – Paper 1



Binary arithmetic

Unit 6
Data types



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Objectives

- Use sign and magnitude to represent negative numbers in binary
- Use two's complement to represent negative numbers in binary
- Add and subtract binary integers
- Represent fractions in fixed point binary

Arithmetic and comparison operators

- All computer instructions are based on the fundamental arithmetic and comparison operators
 - What are these operators?

Arithmetic and comparison operators

- Arithmetic operators:
 - Addition, subtraction, multiplication, division, DIV, MOD, exponentiation
- Comparison operators:
 - Equal, not equal, greater than, less than, greater than or equal, less than or equal (=, <>, >, <>=, <=)

Binary addition

$$1 + 1 =$$

10?

- Computers are built from combinations of electronics called logic gates
- The most fundamental operation a computer processor performs is adding values
- Binary values are input into the processor and

Standardary

- The result of an addition can have a value and a **carry** when the result is too large for the number position it is in
- Working right to left :

1. Add the Units

2. If Over 9, *Carry* Tens

3. Add Tens

$$\begin{array}{r} & 1 \\ & 19 \\ + & 17 \\ = & 36 \end{array}$$



Binary addition

- The rules of binary addition are the same:
 - Three 1s occur during a carry operation

The diagram illustrates four examples of binary addition:

- Example 1:** $0 + 0 = 0$
- Example 2:** $0 + 1 = 1$ (OR $1 + 0 = 1$)
- Example 3:** $1 + 0 = 1$
- Example 4:** $1 + 1 = 10$ (Carry 1)

The carry operation in Example 4 is highlighted with a red '1' above the result.

0	0	1	1	1
$+ 0$	$+ 1$	$+ 0$	$+ 1$	$+ 1$
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
0	1	1	10	11



Adding binary values

- This concept can be extended to binary values of more than one bit

$$\begin{array}{r} \text{Carry} & 1 & 1 & 1 \\ & 1 & 0 & 1 & 0 & 1 \\ + & 1 & 1 & 1 & 1 & 1 \\ \hline = & 1 & 1 & 1 & 0 & 0 \end{array}$$

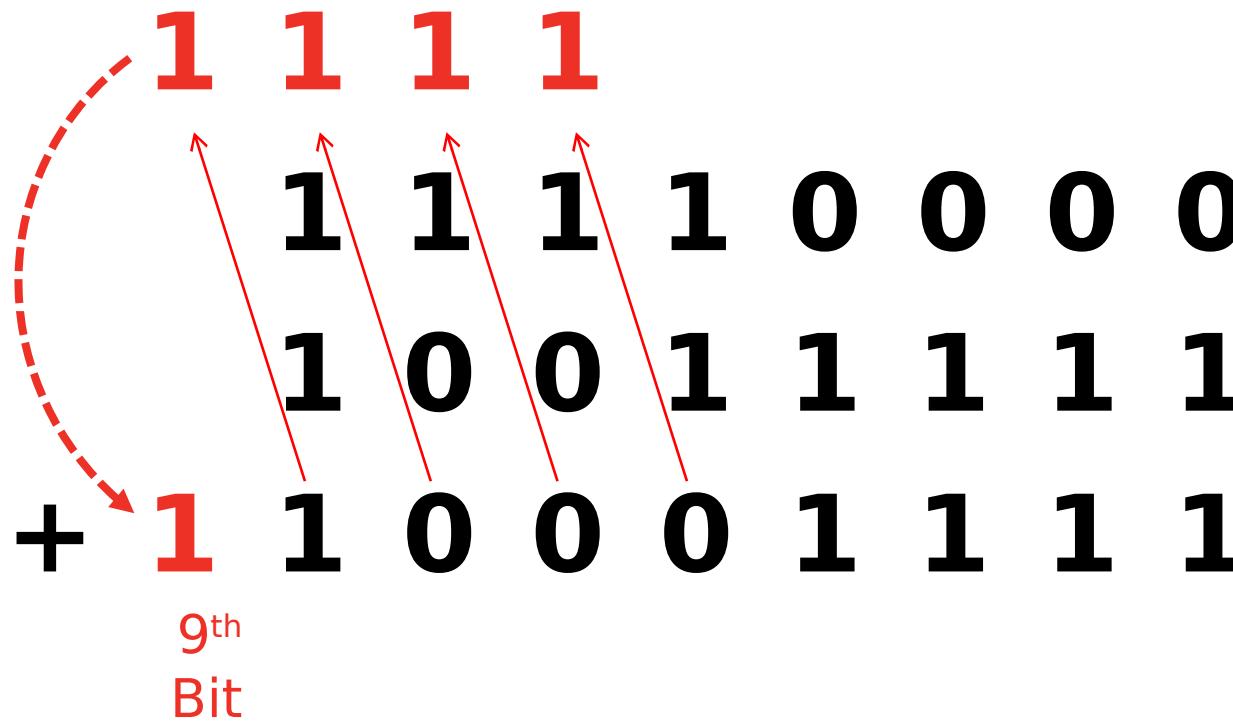
Adding bytes

- Computers work with a fixed number of bits at a time
 - This can cause problems
 - What problem will there be when adding the following bytes and storing the result in one byte?

$$\begin{array}{r} 11110000 \\ +10011111 \\ \hline \end{array}$$

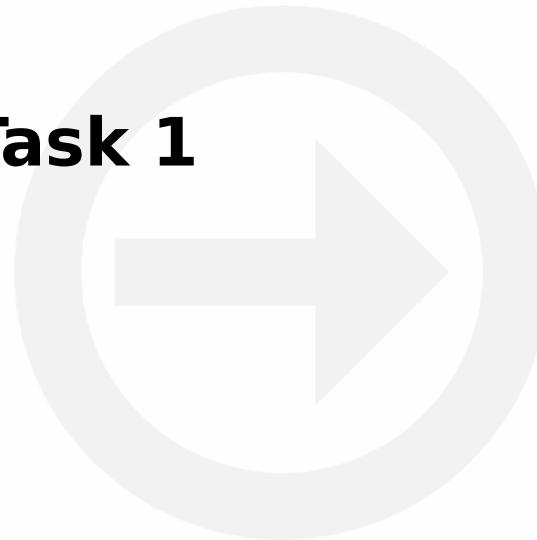
Overflow error

- When the result of an addition is too large for the number of bits the computer works with, there will be an **overflow error**



Binary addition

- Complete **Worksheet 3, Task 1**



Calculating range

- Remember that the number of bits determines how many values can be represented in binary
 - How many binary values can be represented in 4 bits?
 - What is the highest and lowest values that can be represented?

0 . . . 2ⁿ -

1

Representing negative numbers

- How can negative numbers be represented?
- One method is called sign and magnitude
- The first bit acts as the sign, 0 for + and 1 for -
- How would the number -3 be represented in an 8-bit byte using sign and magnitude?

Sign and magnitude

- The binary representation of 3 is 00000011
- The binary representation of -3 is 10000011
- Add these two numbers:

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1 \\ + 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1 \\ \hline = 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \end{array}$$

- This is -6!
- Clearly arithmetic does not work in the normal way using sign and magnitude

Two's complement

- Negative values are most commonly represented using a method called **two's complement**
 - Similar to an analogue counter, turning the wheel back 1 will equal 9999



- Turning back 1 from 00000000 is 11111111
- 11111111 is -1 in binary

Two's complement

- The two's complement of a signed binary value is found by flipping all of the bits and adding one

111111101	=	-3
111111110	=	-2
111111111	=	-1
000000000	=	0
000000001	=	1
000000010	=	2
000000011	=	3



Signed binary representation

• Find -77_{10} in two's complement:

- Find the positive value first:

12 8	64	32	16	8	4	2	1
---------	----	----	----	---	---	---	---

0 1 0 0 1 1 0 1 = 77

- Flip the bits:

1 0 1 1 0 0 1 0

- And add 1:

1 0 1 1 0 0 1 1 = -77



Two's complement range

- Note that the number of values that can be represented is the same as for positive binary numbers
- However, half the range of values now represents negative numbers
 - Maximum for 8 bits is 127_{10} which is 01111111 in binary
 - Minimum is -128_{10} which is 10000000 in binary

$$-(2^{(n-1)}) \dots 2^{(n-1)} - 1$$

- What range of values can be represented in 16 bits?

Subtraction with two's complement

- Two's complement can be used to ensure subtraction will occur when adding negative values
- The calculation $65_{10} - 43_{10}$ should produce the same result as $65_{10} + -43_{10}$
- This way we can always use addition when we want to subtract

Binary subtraction

- Using two's complement we can find $65_{10} + -43_{10}$

 43_{10}

$$\begin{array}{r} \textcolor{red}{1} \textcolor{red}{1} & & & & & & \textcolor{red}{1} \\ & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ + & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ \hline \textcolor{red}{1} & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}$$

*Ignore the
carry bit*

*Answer is
 22_{10}*



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Two's complement overflow

- Where the result of the addition is too large to fit in the allocated number of bits, overflow will occur

- This is the same as standard binary addition
- If the result requires more bits than are available, the number will not be represented correctly
- A '1' in the left most bit indicates a negative number, therefore overflow occurs above 127 with 8 bits

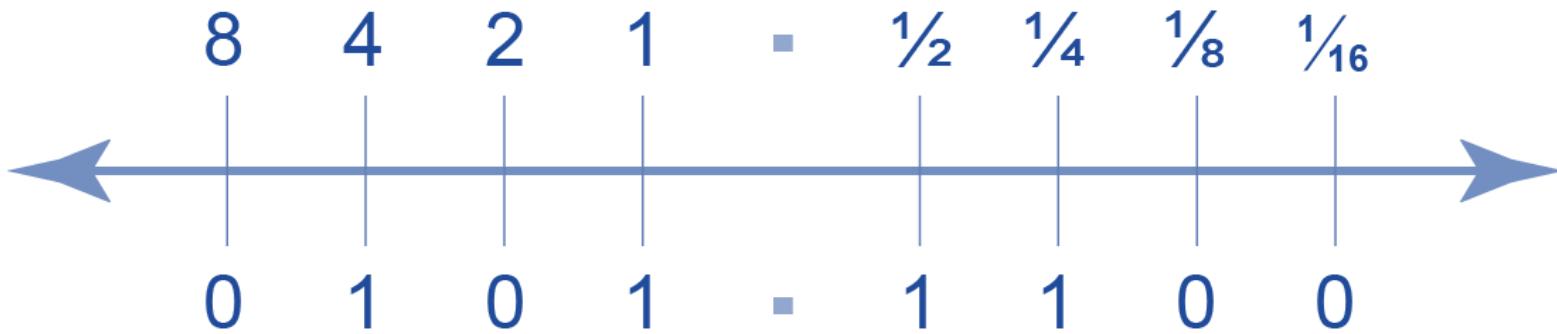
1 1010101

Two's complement

- Complete **Task 2** on **Worksheet 3**

Binary fractions

- Using bits to the right of the units column (after a notional point) introduces fractional values



Fractional values

- Fractional values are negative powers of 2
- For example:
 - 2^{-2} is the same as $\frac{1}{2^2}$ or 0.25_{10}

Place value	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Decimal equivalent	4	2	1	0.5	0.25	$\frac{0.12}{5}$



Fixed-point binary

- A fixed-point binary value uses a specified number of bits where the placement of the binary point is fixed
 - For example, in an 8 bit fixed-point binary value, the binary point could be set between the fourth and fifth bits

2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
8	4	2	1	0.5	0.25	0.125	0.0625
0	1	1	0	1	0	1	0

- What effect does this have on accuracy and range?

Fixed point binary

- Complete **Task 3** on **Worksheet 3**

Plenary

- The fundamental building block of processor operation is addition
- Negative values use two's complement in order to be processed correctly without the need for additional processor logic circuits
- Fixed point binary is one way of representing binary fractions

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